Serie 01 - Solution

Preamble

1.1 Dopant ionization

As seen during the course, the number of holes p_0 and electrons n_0 in a semiconductor are at equilibrium. Most of the interesting effects of the semiconductor arise from the carrier concentration gradient. To change the carrier concentration, we use dopants, which are atoms that are similar in size to the atoms in the semiconductor. Therefore, they can easily substitute them without significantly altering the lattice. Dopants have the ability to more easily give or capture electrons. We refer to this phenomenon of giving or taking electrons as dopant ionization. This phenomenon is temperature-dependent (more information on it in the next series). An approximation that is commonly made, and considered to be true in this course unless specifically denied, is that at room temperature all dopants are ionized.

For example, boron (B) is commonly used as an acceptor dopant in silicon (Si). At low temperatures, the acceptor atoms are not ionized and do not capture any electrons. As the temperature increases, the probability of the acceptor atom capturing an electron also increases. When this happens, the acceptor atom becomes negatively charged as it gets ionized, creating a hole in the valence band that carries a positive charge. This process maintains the overall neutrality of the semiconductor.

An analogous reasoning can be applied to donors. If the doping level or doping concentration of acceptors is denoted as N_a in $\left[cm^{-3}\right]$, the concentration of ionized acceptors is denoted as N_a^-, also in $\left[cm^{-3}\right]$. Similarly, the doping level for donors is denoted as N_d, and the concentration of ionized donors is denoted as N_d^+. Therefore, at room temperature, it is often stated:

$$N_a^- = N_a \qquad \& \qquad N_d^+ = N_d \tag{1}$$

As explained, the doping ionization phenomenon does not alter the overall neutrality of the semiconductor. Therefore, the charge neutrality equation can be written as:

$$\sum Q = p_o + N_d^+ - n_0 - N_a^- = 0 \tag{2}$$

Given constants

$k = 1.3806504 \cdot 10^{-23}\,[J/K]$
$q = 1.6021765 \cdot 10^{-19}\,[C]$

Exercise 01

Calculate the carrier concentration at thermal equilibrium in an n-type semiconductor at $T = 300\,[K]$ doped with $N_d = 10^{16}\,\left[cm^{-3}\right]$. The intrinsic carrier concentration in this semiconductor is $n_i = 1.5 \cdot 10^{10}\,\left[cm^{-3}\right]$.

Solution

As explained in the preamble of this series, all the dopants are considered to be ionized at room temperature. In our case, we don't have any acceptors, so $N_a = 0$.

$$N_d^+ = N_d \tag{3}$$

Therefore, the charge neutrality equation is:

$$\sum Q = n_0 + N_a^- - p_0 - N_d^+ = 0 \tag{4}$$

Since we are in thermal equilibrium, we can write:

$$p_0 = \frac{n_i^2}{n_0} \tag{5}$$

And Therefore:

$$n_0 + N_a^- - \frac{n_i^2}{n_0} - N_d^+ = 0 \tag{6}$$

$$n_0^2 + n_0\left(N_a^- - N_d^+\right) - n_i^2 = 0 \tag{7}$$

By solving this second-degree equation, we obtain:

$$n_0 = \frac{N_d^+ - N_a^- + \sqrt{\left(N_a^- - N_d^+\right)^2 + 4n_i^2}}{2} \approx 1 \cdot 10^{16}\,\left[cm^{-3}\right] \tag{8}$$

We observe that for $N_d^+ \gg n_i$, we have $n_0 \approx N_d^+$. And as we still are in equilibrium:

$$p_0 = \frac{n_i^2}{n_0} \approx 2.25 \cdot 10^4\,\left[cm^{-3}\right] \tag{9}$$

To Go Further

An analogous reasoning as the previous one can be applied, but this time by extracting p_0, which gives us:

$$p_0 = \frac{N_a^- - N_d^+ + \sqrt{\left(N_d^+ - N_a^-\right)^2 + 4n_i^2}}{2} \tag{10}$$

By rearranging the equation, we obtain:

$$p_0 = \frac{N_a^- - N_d^+}{2} + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2 + n_i^2} \tag{11}$$

In this form, three useful approximations can be easily derived. The first case arises when $N_a^- - N_d^+$ is much larger than n_i. The second case occurs when the absolute value of $N_a^- - N_d^+$ is much smaller than n_i. And the last case occurs when $N_d^+ - N_a^-$ is much larger than n_i.

Case 1: If $N_a^- - N_d^+ \gg n_i$, the following approximation can be made:

$$p_0 \approx \frac{N_a^- - N_d^+}{2} + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2} = N_a^- - N_d^+ \tag{12}$$

And since n_0:

$$n_0 \approx \frac{n_i^2}{N_a^- - N_d^+} \tag{13}$$

Case 2: If $\left|N_a^- - N_d^+\right| \ll n_i$, the following approximation can be made from Eq. 11:

$$p_0 \approx \frac{0}{2} + \sqrt{\left(\frac{0}{2}\right)^2 + n_i^2} = n_i \tag{14}$$

And since n_0:

$$n_0 \approx \frac{n_i^2}{n_i} = n_i \approx p_0 \tag{15}$$

Case 3: If $N_d^+ - N_a^- \gg n_i$, an analogous reasoning from **Case 1** can be followed, but with n_0, and in this case, the following approximation can be made:

$$n_0 \approx \frac{N_d^+ - N_a^-}{2} + \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2} = N_d^+ - N_a^- \tag{16}$$

And since p_0:

$$p_0 \approx \frac{n_i^2}{N_d^+ - N_a^-} \tag{17}$$

Summary: The three preceding cases give us the following results: This also gives the following results:

$$p_0 \approx \begin{cases} N_a^- - N_d^+ & ,\text{if} \quad \left(N_a^- - N_d^+\right) \gg n_i \\ n_i & ,\text{if} \quad \left|N_a^- - N_d^+\right| \ll n_i \\ \frac{n_i^2}{N_d^+ - N_a^-} & ,\text{if} \quad \left(N_d^+ - N_a^-\right) \gg n_i \end{cases} \tag{18}$$

$$n_0 \approx \begin{cases} N_d^+ - N_a^- & ,\text{if} \quad \left(N_d^+ - N_a^-\right) \gg n_i \\ n_i & ,\text{if} \quad \left|N_d^+ - N_a^-\right| \ll n_i \\ \frac{n_i^2}{N_d^+ - N_a^-} & ,\text{if} \quad \left(N_a^- - N_d^+\right) \gg n_i \end{cases} \tag{19}$$

This also gives the following results:

$$\left(N_a^- - N_d^+\right) \gg n_i \qquad \Rightarrow \qquad p_0 \gg n_0 \tag{20}$$

$$\left|N_a^- - N_d^+\right| \ll n_i \quad \Rightarrow \quad p_0 \approx n_0 \approx n_i \tag{21}$$

$$\left(N_d^+ - N_a^-\right) \gg n_i \qquad \Rightarrow \qquad n_0 \gg p_0 \tag{22}$$

Exercise 02

Calculate the carrier concentration at thermal equilibrium in a compensated p-type semiconductor at $T = 300\,[K]$ doped with $N_d = 3 \cdot 10^{15}\,\left[cm^{-3}\right]$ and $N_a = 10^{16}\,\left[cm^{-3}\right]$. The intrinsic carrier concentration in this semiconductor is $n_i = 1.5 \cdot 10^{10}\,\left[cm^{-3}\right]$.

Solution

An analogous reasoning as the previous one can be applied in this exercise, but for the sake of simplicity, we will reuse Eq. 8 obtained in the previous exercise.

Once again, as mentioned in the preamble of this series, all dopants are assumed to be ionized at room temperature. Therefore:

$$N_d^+ = N_d \qquad \& \qquad N_a^- = N_a \tag{23}$$

Finally:

$$n_0 = \frac{N_d^+ - N_a^- + \sqrt{\left(N_a^- - N_d^+\right)^2 + 4n_i^2}}{2} \approx 3.21 \cdot 10^4\,\left[cm^{-3}\right] \tag{24}$$

And:

$$p_0 = \frac{n_i^2}{n_0} \approx 7 \cdot 10^{15}\,\left[cm^{-3}\right] \tag{25}$$

Notes: This result can also be directly extracted from the approximation developed in the **To Go Further** section of **Exercise 01**.

Exercise 03

Consider a GaAs semiconductor sample with an intrinsic carrier concentration of $n_i = 2.14 \cdot 10^6 \left[cm^{-3}\right]$ at a temperature of $T = 300\,[K]$. The sample is doped with a donor concentration of $N_d = 10^{16} \left[cm^{-3}\right]$ and no acceptors $N_a = 0 \left[cm^{-3}\right]$. The electron mobility is $\mu_n = 8500 \left[\frac{cm^2}{Vs}\right]$, and the hole mobility is $\mu_p = 400 \left[\frac{cm^2}{Vs}\right]$. Calculate the drift current density for an applied electric field of $E = 10 \left[\frac{V}{cm}\right]$.

Solution

You have seen during the course that the drift current can be calculated with this formula:

$$J^{drift} = q\left(n\mu_n + p\mu_p\right)E \tag{26}$$

Again, since we are at room temperature, we will consider all dopants to be ionized. We can use the Eq. 8 to calculate n_0 and p_0. This gives us the following result:

$$n_0 = \frac{N_d^+ - N_a^- + \sqrt{\left(N_a^- - N_d^+\right)^2 + 4n_i^2}}{2} \approx 1 \cdot 10^{16} \left[cm^{-3}\right] \tag{27}$$

And:

$$p_0 = \frac{n_i^2}{n_0} \approx 5.81 \cdot 10^{-4} \left[cm^{-3}\right] \tag{28}$$

Finally, by substituting this value into Eq. 26:

$$J^{drift} \approx 136 \left[\frac{A}{cm^2}\right] \tag{29}$$

To Go Further

In many semiconductors, although the hole mobility is often much lower than the electron mobility, both remain in the same order of magnitude. On the other hand, the electron and hole concentrations vary by multiple orders of magnitude. Therefore, the following approximation is often made:

$$J^{drift} \approx \begin{cases} qn\mu_n E & ,\text{if} \quad n \gg p \\ qp\mu_p E & ,\text{if} \quad n \ll p \end{cases} \tag{30}$$

Exercise 04

Consider a silicon-based semiconductor bar with a rectangular cross-section at $T = 300\,[K]$ and a doping concentration of $N_d = 5 \cdot 10^{15} \left[cm^{-3}\right]$. We aim to convert this semiconductor bar into a p-type semiconductor by introducing a doping concentration N_a. The resulting semiconductor bar exhibits a resistance of $R = 10\,[k\Omega]$ and a current density of $J = 50\left[\frac{A}{cm^2}\right]$ when a voltage of $V = 5\,[V]$ is applied across the bar, generating an electric field of $E = 100\left[\frac{V}{cm}\right]$ within it.

- Find the length L and the area A of the bar.
- Find the doping concentration N_a.

Solution

- Find the length L and the area A of the bar.

By applying $V = 5\,[V]$ to the bar, we have an internal field of $E = 100\left[\frac{V}{cm}\right]$. Therefore:

$$L = \frac{V}{E} = 0.05\,[cm] = 500\,[\mu m] \tag{31}$$

We also know that the bar has a resistance of $R = 10\,[k\Omega]$, which means that at $V = 5\,[V]$, we have:

$$I = \frac{V}{R} = 500\,[\mu A] \tag{32}$$

And since we want a current density of $J = 50\left[\frac{A}{cm^2}\right]$, we have:

$$A = \frac{I}{J} = 10^{-5}\left[cm^2\right] = 1000\left[\mu m^2\right] \tag{33}$$

- Find the doping concentration N_a.

We extract the desired conductivity σ for our parameters:

$$\sigma = \frac{L}{RA} \approx 0.5\left[\Omega^{-1}cm^{-1}\right] \tag{34}$$

We also know that the conductivity of a semiconductor is defined as:

$$\sigma = q\left(\mu_n n + \mu_p p\right) \tag{35}$$

We want a p-type semiconductor, so we will consider that $p \gg n$. As explained in the **To Go Further** section of **Exercise 03**, we will make the following approximation:

$$\sigma = q\mu_p p \tag{36}$$

And with the previous consideration $p \gg n$, we can also use the approximation developed in the **To Go Further** section of **Exercise 01**:

$$p = N_a^- - N_d^+ \tag{37}$$

And therefore:

$$\sigma = q\mu_p\left(N_a^- - N_d^+\right) \tag{38}$$

Finally, with Fig. 1, we will try some points. For example, with $N_a = 1.25 \cdot 10^{16}\left[cm^{-3}\right]$, we have $\mu_p = 410\left[\frac{cm^2}{Vs}\right]$. This gives us:

$$\sigma = q\mu_p\left(N_a^- - N_d^+\right) \approx 0.493\left[\Omega^{-1}cm^{-1}\right] \tag{39}$$

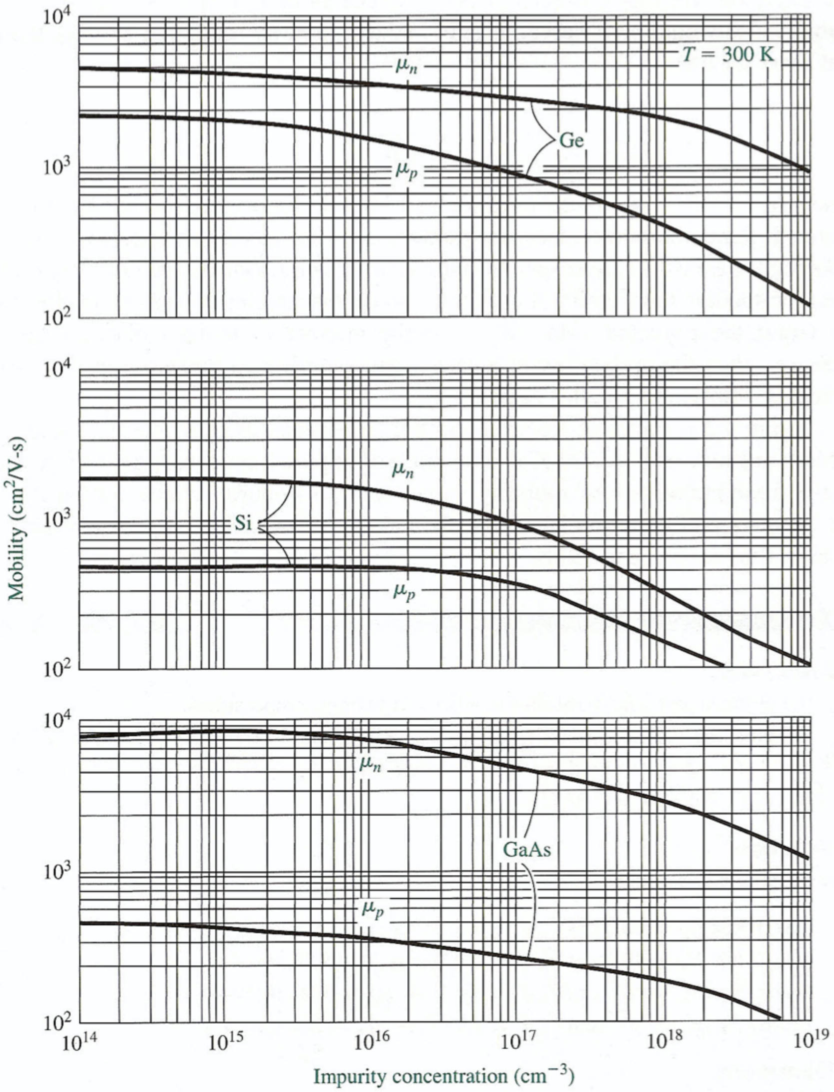

Figure 1: Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300\,[K]$.